The Effect of Tolerances on Microstripline and Slotline Performances

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Abstract—**The change in characteristics of a transmission line due to the tolerances of its parameters has been analyzed. The worst case behavior has been examined using the sensitivity approach. The analysis can be used to determine the tradeoff between performance and tolerances. Computations have been carried out for microstripline and slotline. The fabrication accuracies of linewidth and gapwidth for given tolerances in substrate parameters and specified accuracy in transmission-line characteristics have been evaluated. It has been observed that low-impedance microstriplines and high-impedance slotlines are less sensitive to tolerances in parameters.**

I. Introduction

Microwave integrated circuits (MIC's) are fabricated using a number of techniques, such as photoetching, vacuum deposition, sputtering, electroplating, and screen printing. The fabrication process and the substrate properties, like surface finish, metalization thickness, etc., determine accuracies of fabrication of linewidth and gapwidth, and also determine the minimum obtainable linewidth and gapwidth. In addition to the error in fabrication of linewidth (or gapwidth), the thickness and the dielectric constant of the substrate have some manufacturing tolerances. All these factors contribute to a variation in the characteristic impedance Z_0 and the effective dielectric constant ε_{eff} of the transmission line. The resultant effect on the circuit performance can be analyzed and the substrate tolerances may be specified analytically. Alternatively, the requirements of a fabrication technology suitable for achieving the given circuit specifications may be obtained.

It is necessary to take into account the effect of tolerances at the design stage since postfabrication tuning is almost absent for most of the MIC's. This study will also be useful in the computer-aided design of MIC's. Only a few studies of the effect of tolerances on microstrip circuits have been reported [1], [2].

In this paper a method of evaluating the effect of tolerances on the characteristics of a transmission line in terms of sensitivities has been given. The worst case behavior has been examined. Detailed computations have been presented for the performances of open microstripline and slotline. It is found that the requirements of fabrication technology and/or the specifications of substrate tolerances can be relaxed by using combinations of microstriplines and slotlines.

Manuscript received December 21, 1976; revised May 18, 1977
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II. Analysis

Tolerance analysis by the sensitivity approach is very useful for determining the behavior of a system due to the first-order effects of parameter variations. It is based on small variations in parameter values. The study of worst case behavior, using this approach, requires minimum information on actual changes in the values of parameters [3].

The sensitivity of a parameter A with respect to a parameter B is defined as

$$S_B^A = \operatorname*{Limit}_{\Delta B \to 0} \frac{\Delta A/A}{\Delta B/B} \tag{1}$$

or

$$S_B^A = \frac{B}{A}\frac{\partial A}{\partial B}. \tag{2}$$

The sensitivity, as defined in (1) or (2), may be utilized to determine the deviation in circuit characteristics for a given tolerance in a parameter. The circuit characteristics may be, e.g., VSWR, insertion loss, isolation, and coupling for a branchline coupler. Expressions for the change in the characteristics of a transmission line are developed in the following section. The change in characteristic impedance ΔZ is represented in terms of VSWR performance, which is obtained when the line is connected to a transmission line of exact dimensions. The representation of ΔZ in terms of VSWR is preferred because here it represents the tolerance in Z_0. Change in the effective dielectric constant is also evaluated. Variation in phase velocity may be derived therefrom.

A. Change in the Characteristics of a Transmission Line

The change in impedance ΔZ of a transmission line is related to the tolerance ΔB in parameter B by the relation [using (1)]

$$\frac{\Delta Z}{Z_0} = \frac{\Delta B}{B} S_B^{Z_0} \tag{3}$$

where Z_0 is the impedance with ΔB equal to zero. When Z_0 is a function of several independent variables B_n, $n = 1, 2, \cdots, N$, the total change in Z_0 is, therefore, given by

$$\frac{\Delta Z}{Z_0} = \sum_{n=1}^{N} \frac{\Delta B_n}{B_n} S_{B_n}^{Z_0}. \tag{4}$$

0018-9480/78/0100-0016$00.75

The tolerances ΔB_n will give rise to a number of values for ΔZ. The largest value of ΔZ is related to the worst case behavior, and its value can be obtained from the following relation:

$$\frac{(\Delta Z)_{\max}}{Z_0} = \pm \sum_{n=1}^{N} \left| \frac{\Delta B_n}{B_n} S_{B_n}^{Z_0} \right|. \tag{5a}$$

Similarly, the maximum change $(\Delta\varepsilon_{\text{eff}})_{\max}$ in ε_{eff} is given by

$$\frac{(\Delta \varepsilon_{\text{eff}})_{\max}}{\varepsilon_{\text{eff}}} = \pm \sum_{n=1}^{N} \left| \frac{\Delta B_n}{B_n} S_{B_n}^{\varepsilon_{\text{eff}}} \right|. \tag{5b}$$

Equations (5) can be used to determine the maximum change in Z_0 and ε_{eff} as a function of tolerances.

The alternative problem of determining the requirements of fabrication technology for the permitted change in transmission-line characteristics is considered next.

B. Determination of Fabrication Accuracy

The fabrication accuracy ΔB_m of parameter B_m, for a specified value of ΔZ, is the value of ΔB_m obtained from the following relation:

$$|S_{B_m}^{Z_0}| \frac{|\Delta B_m|}{B_m} = \left| \frac{\Delta Z}{Z_0} \right| - \sum_{\substack{n=1 \cdots N \\ n \neq m}} \left| \frac{\Delta B_n}{B_n} S_{B_n}^{Z_0} \right|. \tag{6a}$$

Equation (6a) holds when the right-hand side is positive. Similarly, for a specified value of $\Delta\varepsilon_{\text{eff}}$, the value of ΔB_m may be obtained from

$$|S_{B_m}^{\varepsilon_{\text{eff}}}| \frac{|\Delta B_m|}{B_m} = \left| \frac{\Delta \varepsilon_{\text{eff}}}{\varepsilon_{\text{eff}}} \right| - \sum_{\substack{n=1 \cdots N \\ n \neq m}} \left| \frac{\Delta B_n}{B_n} S_{B_n}^{\varepsilon_{\text{eff}}} \right|. \tag{6b}$$

When the criteria of both ΔZ and $\Delta\varepsilon_{\text{eff}}$ have to be met, the minimum of two ΔB_m values should be chosen.

III. Application to Microstripline and Slotline

The analysis presented in Section II is useful in determining the tradeoff between tolerances and performance. The requirements of fabrication technology for a specified performance of a transmission line may also be ascertained.

The analysis is general and can be applied to any type of transmission line. Detailed computations are presented for microstripline and slotline only. For these transmission lines, the characteristic impedance and the effective dielectric constant are a function of linewidth or gapwidth W, substrate dielectric constant ε_r, and dielectric thickness d. The effect of metal thickness is very small [4] and has therefore been neglected here. For these lines, (5) become

$$\pm \frac{(\Delta Z)_{\max}}{Z_0} = \left| \frac{\Delta W}{W} S_W^{Z_0} \right| + \left| \frac{\Delta d}{d} S_d^{Z_0} \right| + \left| \frac{\Delta \varepsilon_r}{\varepsilon_r} S_{\varepsilon_r}^{Z_0} \right| \tag{7a}$$

$$\pm \frac{(\Delta \varepsilon_{\text{eff}})_{\max}}{\varepsilon_{\text{eff}}} = \left| \frac{\Delta W}{W} S_W^{\varepsilon_{\text{eff}}} \right| + \left| \frac{\Delta d}{d} S_d^{\varepsilon_{\text{eff}}} \right| + \left| \frac{\Delta \varepsilon_r}{\varepsilon_r} S_{\varepsilon_r}^{\varepsilon_{\text{eff}}} \right|. \tag{7b}$$

It is simple to obtain similar results for other types of transmission lines using dielectric substrates; e.g., stripline, suspended-substrate microstripline, coplanar line, inverted-strip dielectric waveguide, etc.

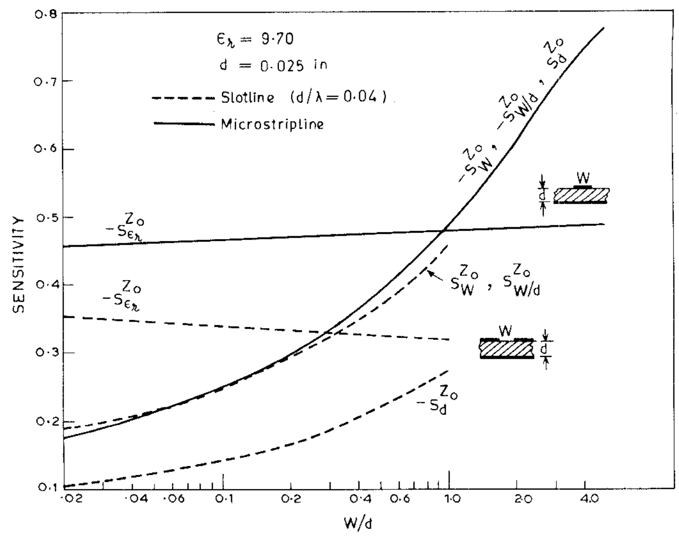

Fig. 1. Sensitivity of the characteristic impedance of microstripline and slotline to its parameters.

A. Evaluation of Sensitivities

Since the definition of sensitivity involves partial derivative of Z_0 and ε_{eff} with respect to one of the parameters, expressions for sensitivities can be obtained if closed-form expressions relating Z_0 and ε_{eff} with W, d, and ε_r are available. For slotline, Garg and Gupta have reported closed-form expressions [5] based on Cohn's analysis [6]. In the case of microstripline, the expressions obtained by Schneider for the quasi-TEM mode have been used [7]. The accuracy of these relations is about 2 percent [5], [8]. The effect of dispersion in the microstrip case can be incorporated by using appropriate expressions for Z_0 and ε_{eff}. For the dispersive behavior of ε_{eff}, one can use relations reported by Edwards and Owens [9]. The dispersive behavior of Z_0 is also described by a similar relation given by Bianco *et al.* [10].

The sensitivities of microstripline and slotline impedances and effective dielectric constants with respect to their parameters are plotted in Figs. 1 and 2, respectively. The value of the dielectric constant chosen is 9.7. The following points may be observed from Figs. 1 and 2.

1) The sensitivity curves for Z_0 and ε_{eff} have smooth variations.

2) $S_W^{Z_0}$ is negative for microstripline and positive for slotline. This is due to the fact that the impedance for a microstripline decreases with an increase in the linewidth. The reverse is true for a slotline.

3) For a microstripline, the sensitivity curves for W and d coincide and have different signs. This occurs because W and d appear as a single variable W/d in the expression for Z_0 [7] and the definition of sensitivity involves a first derivative with respect to the variable.

4) High-impedance microstriplines ($W < d$) are more sensitive to tolerance in ε_r than to tolerances in W and d, since

$$|S_{\varepsilon_r}^{Z_0}| > |S_W^{Z_0}| \quad \text{or} \quad S_d^{Z_0}.$$

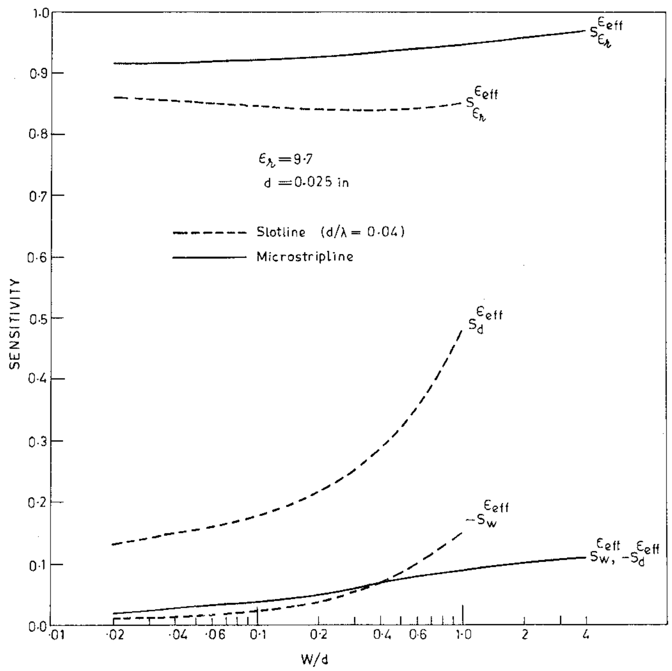

Fig. 2. Sensitivity of the effective dielectric constant of microstripline and slotline to its parameters.

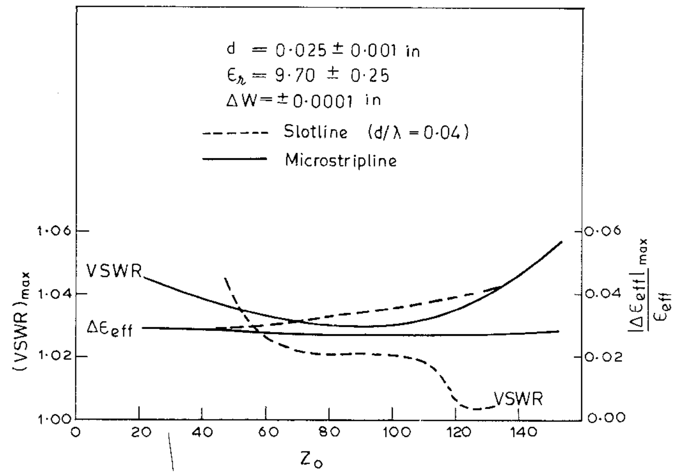

Fig. 3. Effect of dimensional tolerances on the change in characteristics.

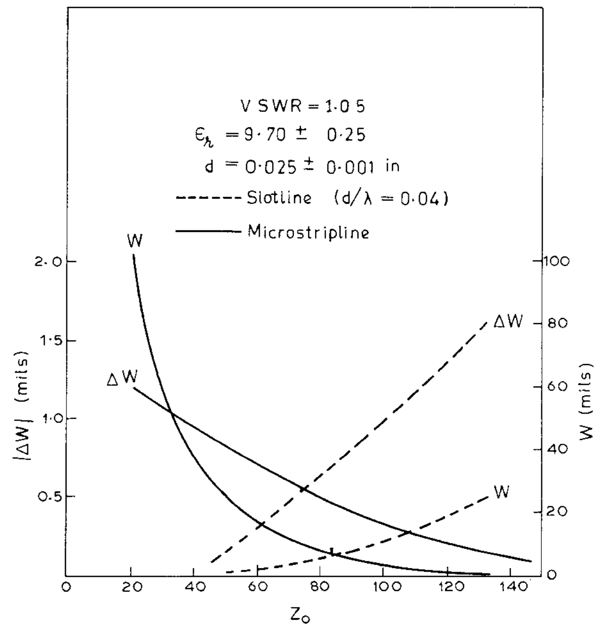

Fig. 4. Fabrication accuracy requirements for the given substrate tolerances and the required VSWR performance.

5) Low-impedance microstriplines ($W > d$) are more sensitive to variations in W and d as compared to variations in ε_r.

6) $S_{\varepsilon_r}^{Z_0}$ for microstripline is almost constant with Z_0, and its average value is 0.475 for $\varepsilon_r = 9.7$.

7) Slotline is relatively less sensitive than microstripline to changes in W, d, and ε_r.

Similar observations can also be made about sensitivity curves for ε_{eff} (Fig. 2).

The values of sensitivities by themselves are not sufficient to determine the change in the characteristics of transmission line or to predict the constraints on tolerances of various parameters to improve the performance of transmission line. The contribution of each of these sensitivities to the change in the characteristics also depends on the weight function $\Delta B_n/B_n$, as indicated in (5). Therefore, the sensitivity curves of Figs. 1 and 2 have been utilized along with (7) to plot the maximum absolute change in the characteristics. The minimum change is zero and corresponds to zero tolerances. This is shown in Fig. 3. The fabrication accuracy of the stripwidth or gapwidth has been assumed to be 2.54 μm (0.1 mil). The assumed tolerances in d and ε_r correspond to a commercially available substrate using 99.5-percent pure alumina. It is observed from Fig. 3 that the phase velocity in microstripline is less sensitive than that in slotline. Also high-impedance slotlines and low-impedance microstriplines can be easily realized to have low values of VSWR. Therefore, for low values of VSWR a composite structure of low-impedance microstripline and high-impedance slotline may be preferred compared to all microstripline circuits. This configuration will be useful for narrow frequency-band applications because of the dispersive nature of slotline. However, a slotline provides other advantages, like shunt mounting of devices, series stubs, easily accessible short circuits, reduced size, etc.

B. Fabrication Accuracy

In general, the tolerance in the dielectric thickness and dielectric constant is specified by the manufacturer, and it is desirable to know the permitted tolerance in stripwidth or gapwidth W for specified values of ΔZ (or VSWR) and $\Delta\varepsilon_{\text{eff}}$.

The tolerance or fabrication accuracy of W for a specified value of ΔZ can be obtained from (6a) and is given below

$$|S_W^{Z_0}| \frac{|\Delta W|}{W} = \left|\frac{\Delta Z}{Z_0}\right| - \left|\frac{\Delta d}{d} S_d^{Z_0}\right| - \left|\frac{\Delta \varepsilon_r}{\varepsilon_r} S_{\varepsilon_r}^{Z_0}\right|. \tag{8}$$

The fabrication accuracy ΔW obtained from (8) has been plotted in Fig. 4 as a function of Z_0 for a maximum VSWR value of 1.05. The substrate tolerances are again those of 99.5-percent alumina substrates. The value of ΔW for a specified value of $\Delta\varepsilon_{\text{eff}}$ can be obtained in a similar manner from (6b). When the criteria of both VSWR and $\Delta\varepsilon_{eff}$ have to be met, the minimum of the two ΔW values should be chosen. The fabrication accuracy of d or tolerance in ε_r can be determined in a similar manner, provided the tolerances for the other two parameters are given.

It may be observed from Fig. 4 that the fabrication accuracy for a 50-Ω microstripline should be about 3 percent for a maximum VSWR value of 1.05. Also the fabrication tolerance for a slotline with an impedance greater than 75 Ω is larger than that for the microstripline. The larger fabrication tolerance for the slotline is mainly due to the lower sensitivity of slotline impedance to variations in d and ε_r (Fig. 1). The relatively higher slotline width also contributes to impedances greater than 75 Ω. It may also be seen that the fabrication tolerance limit, for the given specifications of d and ε_r, may be increased to about 0.56 mil (about 14 μm) if microstripline is used for impedances lower than 75 Ω and slotline is used for higher impedance values.

A similar analysis has been carried out for microstripline and slotline on other types of substrates (ε_r and d different). For example, for a 50-Ω microstripline on a polystyrene substrate, with $\varepsilon_r = 2.55 \pm 0.2$, $d = 0.062 \pm 0.005$ in, and a fabrication accuracy of 0.003 in, the maximum value of VSWR obtained is 1.11 and the maximum absolute change in ε_{eff} is 0.16. In that case, it has been observed by Garg [11] that the dimensional tolerances give rise to the largest change in characteristics compared to various other factors, viz., strip thickness ($t/h = 0.2$), dispersion ($f = 6$ GHz), step discontinuity ($W_2/W_1 = 2.0$), launchers, etc.

The analysis reported in this paper may also be utilized to determine the accuracy required of expressions relating the characteristics of the transmission line to its parameters. For example, if the specified set of tolerances gives rise to a value of 1.03 for VSWR, the impedance expression need not have an accuracy better than 3 percent.

Acknowledgment

The author wishes to thank Prof. K. C. Gupta for comments on this paper and for many helpful discussions.

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